A Problem on Fibre Reinforced Micropolar Thermoelastic Half Space at an Elastic Interface

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Abstract: The present study deals with the two dimensional deformation of fibre reinforced micropolar thermoelastic medium. A mechanical force is applied along the interface of elastic half space and fibre reinforced micropolar thermoelastic half space. The analytic expresssions for the considered variables have been obtained using normal mode analysis technique. The effect of microrotation on the derived components have been depicted graphically.

Keywords: fibre-reinforced, thermoelasticity, elastic space.

Introduction

The dynamical interaction between the thermal and mechanical has great practical applications in modern aeronautics, astronatics, nuclear reactors, and high-energy particle accelerators. Classical elasticity is not adequate to model the behavior of materials possessing internal structure. Furthermore, the micropolar elastic model is more realistic than the purely elastic theory for studying the response of materials to external stimuli. Green and Lindsay [1}developed the theory of thermoelasticity. Eringen and Suhubi [2] and Suhubi and Eringen [3] developed a nonlinear theory of micro-elastic solids. Later Eringen [4-6] developed a theory for the special class of micro-elastic materials and called it the "linear theory of micropolar elasticity". Under this theory, solids can undergo macro-deformations and micro-rotations.

A reinforced structural component is designed for all conditions of stresses that may occur and in accordance with the principles of applied mechanics. Fiber-reinforced composites are used in a variety of structures due to their low weight and high strength. A reinforced medium plays a very interesting role in civil engineering and geophysics, as well as aerospace structural dynamics (wings, fuselage etc). The idea of introducing a continuous self-reinforcement at every point of an elastic solid was given by Belfied et al.[7]. The model was later applied to the rotation of a tube by Verma and Rana [8]. Sengupta and Nath [9] discussed the problem of the surface waves in fiber-reinforced anisotropic elastic media. The problem of wave propagation in thermally-conducting linear fibre-reinforced composite materials was analyzed by Singh [10]. Abbas and Othman [11] and Othman and co-workers [12-15] discussed some problems in fiber-reinforced thermoelastic medium. The field equations and constitutive relations for generalized thermo-elastic medium in the absence of body forces, body couples and heat sources in the context of generalized thermo-elasticity are given by [7]:

$$\rho \ddot{u}_j = \sigma_{ij,i}, (i, j = 1,2,3) \qquad (1)$$

$$K^* T_{,ii} = (n_1 \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}) \rho C_E T + \upsilon T_0 (n_1 \frac{\partial}{\partial t} + n_0 \tau_0 \frac{\partial^2}{\partial t^2}) u_{i,i} \quad (2) \quad (\alpha_l + \beta_l + \gamma)\nabla(\nabla.\vec{\varphi}) - \gamma \nabla \times (\nabla \times \vec{\varphi}) + k(\nabla \times \vec{u}) - 2k\vec{\varphi} = J\rho \frac{\partial^2 \vec{\varphi}}{\partial t^2} \quad (3)$$

$$m_{il} = \alpha_l \varphi_{r,r} \delta_{il} + \gamma \varphi_{l,i} \qquad (4)$$

The equations of elastic medium are:

$$(\lambda^e + \mu^e)\nabla(\nabla.\vec{u}^e) + \mu^e \nabla^2 \vec{u}^e = \rho^e \frac{\partial^2 \vec{u}^e}{\partial t^2}, \qquad (5)$$

$$\sigma^e_{ij} = \lambda^e (\nabla . u^e)\delta_{ij} + \mu^e (u^e_{i,j} + u^e_{j,i}). \qquad (6)$$

where, the notations have usual meanings.

we restrict our analysis to the plane strain parallel to xy-plane with displacement vector $\vec{u} = (u, v, 0)$.

Solution of the Problem

We use normal mode analysis technique as,

$$(u,v,T,\sigma_{ij},u^e,v^e,\sigma_{ij}^e)(x,y,t) = (u^*,v^*,T^*,\sigma_{ij}^*,u^e,v^{e^*},\sigma_{ij}^{e^*})(x)e^{\omega t+iby}$$

where ω is complex frequency, b is wave number in y -direction and $u^*(x),v^*(x),T^*(x),\sigma_{ij}^*(x),u^{e^*}(x),v^{e^*}(x),\sigma_{ij}^{e^*}(x)$ are the amplitudes of field quantities.

$$(h_{11}D^2 - A_1)u^* + ibh_2Dv^* + ibh_3\varphi_3^* - A_4DT^* = 0 \quad (7)$$

$$ibh_2Du^* + (h_1D^2 - A_2)v^* - h_3D\varphi_3^* - ibA_4T^* = 0 \quad (8)$$

$$-ibb_1u^* + b_1Dv^* + (D^2 - A_3)\varphi_3^* = 0 \quad (9)$$

$$A_6Du^* + ibA_6v^* - (D^2 - A_5)T^* = 0 \quad (10)$$

$$m_{yz}^* = ibb_4\varphi_3^* \quad (11)$$

$$\rho c_1^2\sigma_{xx}^* = A_{11}Du^* + ibA_{12}v^* - \rho c_1^2A_4T^* \quad (12)$$

$$\rho c_1^2\sigma_{xy}^* = \mu_L(ibu^* + Dv^*) - k\varphi_3^* \quad (13)$$

where, $D \equiv \frac{d}{dx}$, $(h_{11},h_{22},h_1,h_2) = \dfrac{(A_{11},A_{22},B_1,B_2)}{\rho c_1^2}$, $B_1 = \mu_L$,

$A_{11}=\lambda+2(\alpha+\mu_T)+4(\mu_L-\mu_T)+\beta$, $B_2 = A_{12}+B_1$, $A_{12} = \alpha+\lambda$,

$A_{22}=\lambda+2\mu_T$, $A_1=\omega^2+h_1b^2$, $A_2=\omega^2+h_{22}b^2$, $A_3=b^2+b_2-b_3b^2$,

$A_4=1+v_0\omega$, $A_5=b^2+\tau_0\omega^2+n_1\omega$, $A_6=\varepsilon\omega(n_1+n_0\tau_0\omega)$,

$\varepsilon = \dfrac{\vartheta^2T_0}{K^*\eta\rho c_1^2}$, $(b_1,b_2)=\dfrac{(k,2k)}{(\gamma\eta^2c_1^2)}$ $b_3 = \dfrac{J\rho c_1^2}{\gamma}$, $b_4 = \dfrac{\gamma\eta^2}{\rho}$.

Eliminating $v^*(x)$, $T^*(x)$, $\varphi^*(x)$ from (7)-(10), we get

$$(D^8 + AD^6 + BD^4 + CD^2 + E)u^*(x) = 0 \quad (14)$$

$$A=\frac{-1}{h_{11}h_1}(h_1A_4A_6+h_{11}h_1A_5+h_{11}h_1A_3+h_{11}A_2+b_1h_{11}h_3+h_1A_1+ibh_2^2)$$

$$B=\frac{-1}{h_{11}h_1}(2b^2h_2A_4A_6-h_1A_3A_4A_6+b_1h_3A_4A_6-A_2A_4A_6$$
$$+ib^2h_{11}A_4A_6-h_{11}h_1A_3A_5-h_{11}A_2A_5+h_{11}b_1h_3A_5-h_1A_1A_5-ib^2h_2^2A_5$$
$$-h_{11}A_2A_3-h_1A_3A_1-A_1A_2+b_1h_3A_1-ibh_2^2A_3+b^2b_1h_2h_3+b^2b_1h_1h_3)$$

$$C=\frac{-1}{h_{11}h_1}(-2b^2h_2A_3A_4A_6-b^2b_1h_3A_4A_6+A_2A_3A_4A_6-ib^2A_1A_4A_6-ib^2h_{11}A_3A_4A_6+h_{11}A_2A_3A_5+h_1A_1A_3A_5+A_1A_2A_5-b_1h_3A_1A_5$$
$$+ib^2h_2^2A_3A_5-2b^2b_1h_2h_3A_5-b^2b_1h_1h_3A_5+A_1A_2A_3-b^2b_1h_3A_2)$$

$$E=\frac{-1}{h_{11}h_1}(b^4b_1h_3A_4A_6+ib^2A_1A_3A_4A_6-A_1A_2A_3A_5+b^2b_1h_3A_2A_5)$$

The series solution of (14) is given by,

$$u^*(x) = \sum_{n=1}^{4}[M_n(b,\omega)e^{-k_nx}] \quad (15)$$

$$v^*(x) = \sum_{n=1}^{4}[H_{1n}M_n(b,\omega)e^{-k_n x}] \quad (16)$$

$$T^*(x) = \sum_{n=1}^{4}[H_{2n}M_n(b,\omega)e^{-k_n x}] \quad (17)$$

$$m_{yz}^*(x) = \sum_{n=1}^{4}[H_{3n}M_n(b,\omega)e^{-k_n x}] \quad (18)$$

$$\sigma_{xx}^*(x) = \sum_{n=1}^{4}[H_{4n}M_n(b,\omega)e^{-k_n x}] \quad (19)$$

$$\sigma_{xy}^*(x) = \sum_{n=1}^{4}[H_{5n}M_n(b,\omega)e^{-k_n x}] \quad (20)$$

$M_n(b,\omega)$ is specific function depending upon b and ω, k_n^2 ; n =1,2,3,4 are the roots of (14).

$$H_{1n} = \frac{[h_{11}k_n^5 - (A_1 + b^2h_2 + A_5h_{11} + A_4A_6)k_n^3 + (A_1A_5 + b^2h_2A_5 + A_4A_6b^2)k_n]}{ib[(h_1h_2)k_n^4 - (A_2 + h_1A_5 + h_2A_5 + A_4A_6)k_n^2 + (A_2A_5 + b^4A_4A_6)]}$$

$$H_{2n} = \frac{(k_n - ibH_{1n})}{(k_n^2 + A_5)},\ H_{3n} = ibb_4H_{2n},$$

$$H_{4n} = \frac{[-A_{11}k_n + ibA_{12}H_{1n} - \rho c_1^2A_4H_{3n}]}{\rho c_1^2}\ H_{5n} = \frac{[ib\mu_L - k_n\mu_L H_{1n} - kH_{2n}]}{\rho c_1^2}$$

Similarly for elastic half space, the solutions are

$$u^{e^*}(x) = \sum_{n=1}^{2}[R_n(b,\omega)e^{r_n x}],\ v^{e^*}(x) = \sum_{n=1}^{2}[L_{1n}R_n(b,\omega)e^{r_n x}]$$

$$\sigma_{xx}^{e^*} = \sum_{n=1}^{2}[L_{2n}R_n(b,\omega)e^{r_n x}],\ \sigma_{xy}^{e^*} = \sum_{n=1}^{2}[L_{3n}R_n(b,\omega)e^{r_n x}]$$

Where, $R_n(b,\omega)$ is specific function depending upon b and ω and r_n^2; n =1,2 are roots of, $(D^4 - GD^2 + L)u^{e^*}(x) = 0$ (1)

where $G = \dfrac{b^2l_1^2 + l_1\omega^2 + b^2l_3^2 + l_3\omega^2 - b^2l_2^2}{l_1l_3}$

$$L = \frac{b^4l_1l_3 + b^2l_3\omega^2 + b^2l_1\omega^2 + \omega^4}{l_1l_3},\ l_1 = \frac{\lambda^e + 2\mu^e}{\rho^e c_1^2},\ l_2 = \frac{\lambda^e + \mu^e}{\rho^e c_1^2},\quad l_3 = \frac{\mu^e}{\rho^e c_1^2},\ L_{1n} = \frac{(b^2l_3 + \omega^2) - l_1r_n^2}{ibl_2r_n},$$

$$L_{2n} = \frac{(\lambda^e + 2\mu^e)(r_n) + ibL_{1n}}{\rho^e c_1^2},\ L_{3n} = \frac{ib\mu^e + r_nL_{1n}}{\rho^e c_1^2}$$

Boundary Conditions

To determine M_n; (n =1,2,3,4) and R_n ;(n =1,2)., The boundary conditions at the surface $x = 0$ have been taken as,

$\sigma_{xx} = \sigma_{xx}^e - F_1e^{\omega t + iby}$; $\sigma_{xy} = \sigma_{xy}^e$; $m_{yz} = 0$; $u = u^e$; $v = v^e$; $\dfrac{\partial T}{\partial x} = 0$, where F_1 is the magnitude of mechanical force.

Using the expressions of σ_{xx}, σ_{xx}^e, σ_{xy}, σ_{xy}^e, u, u^e, v, v^e, T, m_{yz} into above boundary conditions, we get,

$$\sum_{n=1}^{4}[H_{4n}M_n]-\sum_{n=1}^{2}[L_{2n}R_n]=-F_1,$$

$$\sum_{n=1}^{4}[H_{5n}M_n]-\sum_{n=1}^{2}[L_{3n}R_n]=0,\ \sum_{n=1}^{4}[H_{3n}M_n]=0$$

$$\sum_{n=1}^{4}[M_n]-\sum_{n=1}^{2}[R_n]=0,\ \sum_{n=1}^{4}[H_{1n}M_n]-\sum_{n=1}^{2}[L_{1n}R_n]=0,\ \sum_{n=1}^{4}[H_{2n}k_nM_n]=0$$

After solving these, we get the values of constants $M_1, M_2, M_3, M_4, R_1, R_2$ and hence obtain the components of considered physical quantities.

Numerical Results and Discussions

For numerical computations, numerical values for the physical constants are given by[13]:

$\lambda = 9.4\times10^{9}\,N/m^2$, $\mu_T = 1.89\times10^{9}\,N/m^2$, $T_0 = 274.5K$

$\mu_L = 2.45\times10^{9}\,N/m^2$, $\alpha = -1.28\times10^{9}\,N/m^2$,

$\beta = 0.32\times10^{9}\,N/m^2$, $C_E = 5J/(kgK)$, $k = 10^{11}\,N/m^2$, $\gamma = 0.779\times10^{-1}\,N$, $K^* = 0.3W/(mk)$, $\alpha_t = 7.4033\times10^{-7}k^{-1}$,

$\lambda^e = 2.4\times10^{10}\,N/m^2$, $\mu^e = 1.2\times10^{10}\,N/m^2$, $\rho = 1.7\times10^{3}\,kg/m^3$.

$t = 0.4$ $0\le x\le 10$ $y = 1.3$.The variations are shown in figures (1)-(3) for $F_1 = 1.0$, $\omega = \omega_0 + \iota\xi$, $\omega_0 = -0.3$, $\xi = 0.1$ and (for G-L theory) by taking $\tau_0 = 0.4$,

Disussion and Conclusion

From fig-(1)-(3), it is clear that, normal displacement, normal force stress and tangential couple stress have higher values near the point of application of source, which decreases with the increase in horizontal distance. The values are higher for TS as compared to FRMTS, MTS and FRTS, showing the appreciable effect of mechanical source and anisotropy in a fibre reinforced micropolar thermoelastic medium.

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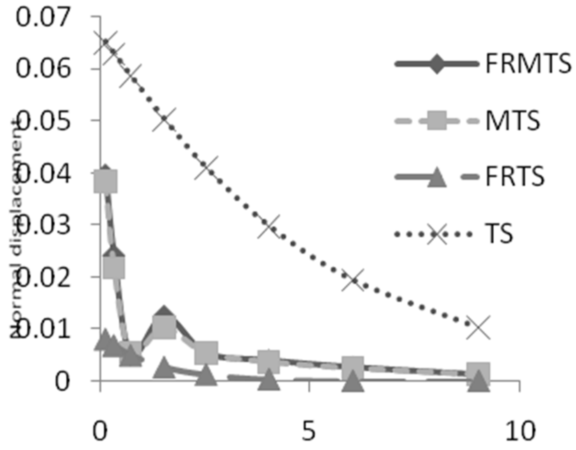

Figure-1: Variation of normal displacement with distance

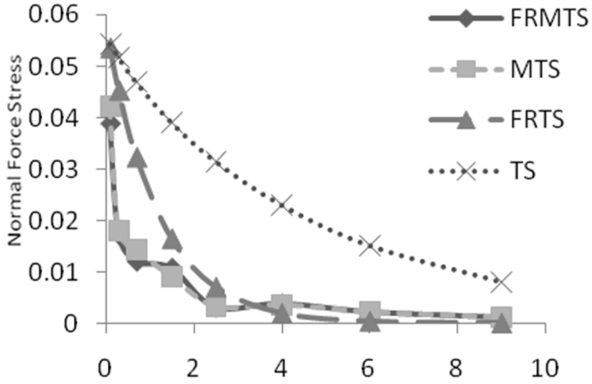

Figure-2: Variation of normal force stress with distance

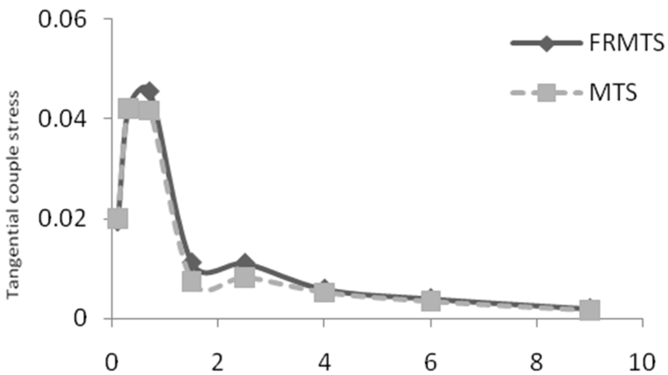

Figure-3: Variation of tangential couple stress with distance